A Moment Method Analysis of a Folded E-Plane Short in Rectangular Waveguide

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Abstract—**A folded E-plane waveguide short is analyzed using a moment method formulation. A comparison of the numerical convergence of the method is shown for two different sets of basis functions: trigonometric aperture waveguide modes and weighted Gegenbauer polynomials. The theory is verified by measurements of a folded short in half-height X-band rectangular waveguide.**

I. Introduction

BECAUSE OF LIMITED space in certain applications where waveguide circuitry is used, it is sometimes necessary to use a folded waveguide short (see Fig. 1) instead of a standard waveguide short. One such example is the feeding networks of large planar slotted waveguide arrays, where resonant waveguide sticks with coupling slots need to be shorted at one end. The position of the short needs to be at a very specific distance (typically half a guide wavelength) from the last coupling slot in the waveguide, and if space does not allow for physical realization of a standard waveguide short, a folded short with equivalent characteristics can be used. Although single and multiple E-plane steps and discontinuities in rectangular waveguide [1], [2] have already been the subject for using rigorous analysis techniques, the folded E-plane waveguide short, to the author's knowledge, has not been investigated by the moment method or mode-matching techniques. This letter presents results of an integral equation formulation moment method analysis of a folded E-plane short in rectangular waveguide. A comparison of the numerical convergence of the method is shown for two different sets of basis functions: trigonometric aperture waveguide modes and weighted Gegenbauer polynomials (also used in [1], [2]). The Gegenbauer polynomials are pre-conditioned to satisfy the expected edge-condition singularity of the normal component of the electric field at a 90° conducting edge. The theory is verified by measurements of a folded short in half-height X-band rectangular waveguide.

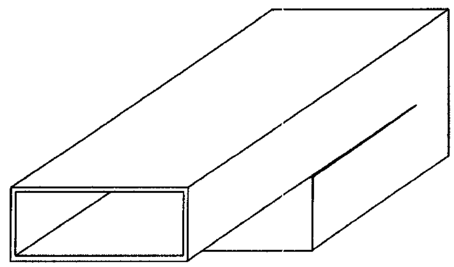

Fig. 1. A folded E-plane waveguide short.

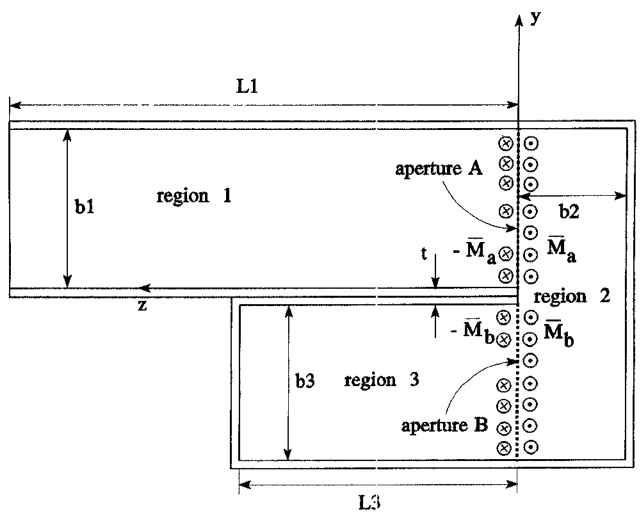

Fig. 2. A cross-section of a folded E-plane waveguide short: the two short circuits with unknown magnetic currents, as used in the analysis, are shown.

II. Moment Method Analysis

The formulation used is based on that in [3]. Using field-equivalence principles, the problem of a waveguide feeding a folded-short can be transformed to the relatively simple problem of three separate but connected regions with fully known characteristics. Fig. 2 shows the folded short divided into three regions (one waveguide section and two waveguide cavities) by replacing two waveguide apertures with short-circuits containing unknown impressed magnetic currents on both sides of each short circuit. The only form of excitation in the problem is the incident field of the dominant TE_{10} mode propagating in the waveguide, and because no x-directed electric fields will be generated by this particular type of discontinuity, the appropriate set of waveguide and cavity modes to use in the analysis will be the LSE_{1n} or TE_{x1n} modes. Applying the continuity conditions for the magnetic fields across the two chosen apertures, and expressing the magnetic fields in terms of the magnetic type Green's functions of the respective regions, two coupled integral equations are obtained. These two equations—with the magnetic currents as unknowns—can be solved using the moment method.

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After expanding the unknown currents as a finite number of expansion functions with unknown coefficients and selecting suitable testing functions and an inner product, the pair of integral equations is converted into a matrix equation of the form

$$\begin{bmatrix}[A_{is}] & [B_{is}]\\ [C_{is}] & [D_{is}]\end{bmatrix}\begin{bmatrix}[a_s]\\ [b_s]\end{bmatrix} = \begin{bmatrix}[h_i]\\ [0]\end{bmatrix} \tag{1}$$

where each of the submatrices are

$$A_{is} = j\omega\epsilon_0 \int\int_{s_a} g_a^i(\overline{r}) \int\int_{s_a} G_{xx}^1(\overline{r}/\overline{r}_o) f_a^s(\overline{r}_o) ds_a ds_a - j\omega\epsilon_0 \int\int_{s_a} g_a^i(\overline{r}) \int\int_{s_a} G_{xx}^2(\overline{r}/\overline{r}_o) f_a^s(\overline{r}_o) ds_a ds_a \tag{2}$$

$$B_{is} = -j\omega\epsilon_0 \int\int_{s_a} g_a^i(\overline{r}) \int\int_{s_b} G_{xx}^2(\overline{r}/\overline{r}_o) f_b^s(\overline{r}_o) ds_b ds_a \tag{3}$$

$$C_{is} = -j\omega\epsilon_0 \int\int_{s_b} g_b^i(\overline{r}) \int\int_{s_a} G_{xx}^2(\overline{r}/\overline{r}_o) f_a^s(\overline{r}_o) ds_a ds_b \tag{4}$$

$$D_{is} = -j\omega\epsilon_0 \int\int_{s_b} g_b^i(\overline{r}) \int\int_{s_b} G_{xx}^2(\overline{r}/\overline{r}_o) f_b^s(\overline{r}_o) ds_b ds_b + j\omega\epsilon_0 \int\int_{s_b} g_b^i(\overline{r}) \int\int_{s_b} G_{xx}^3(\overline{r}/\overline{r}_o) f_b^s(\overline{r}_o) ds_b ds_b \tag{5}$$

$$h_i = -\int\int_{s_a} g_a^i(\overline{r}) H_{xi}(\overline{r}) ds_a \tag{6}$$

with G_{xx}^1, G_{xx}^2 and G_{xx}^3 the xx-components of the dyadic Green's functions of the magnetic type of regions 1, 2, and 3 (see Fig. 2), respectively, H_{xi} the x-component of the dominant incident TE_{10} waveguide mode, and s_a and s_b the surfaces of apertures A and B (see Fig. 2). f_a^s and g_a^i are the expansion and testing functions for the current in aperture A, and f_b^s and g_b^i the expansion and testing functions for the current in aperture B. The analysis was done for two classes of basis functions for the unknown current, namely trigonometric aperture waveguide modes and weighted Gegenbauer polynomials. The weighted Gegenbauer expansion (only even polynomials were used) for the unknown currents, and the testing functions in the moment method formulation, are

$$\overline{M}_a(\overline{r}) = \sum_{s=a}^{N,2} a_s f_a^s(\overline{r})\hat{x} = \sum_{s=0}^{N,2} a_s \sin\left(\frac{\pi x}{a}\right)(1-\eta_a^2)^{-\frac{1}{3}} C_s^{(1/6)}(\eta_a)\hat{x} \tag{7}$$

$$\overline{M}_b(\overline{r}) = \sum_{s=0}^{N,2} b_s f_b^s(\overline{r})\hat{x} = \sum_{s=0}^{N,2} b_s \sin\left(\frac{\pi x}{a}\right)(1-\eta_b^2)^{-\frac{1}{3}} C_s^{(1/6)}(\eta_b)\hat{x} \tag{8}$$

$$g_a^i(\overline{r}) = \sin\left(\frac{\pi x}{a}\right)(1-\eta_a^2)^{-\frac{1}{3}} C_i^{(1/6)}(\eta_a), \quad i = 0, 2, 4, \ldots N \tag{9}$$

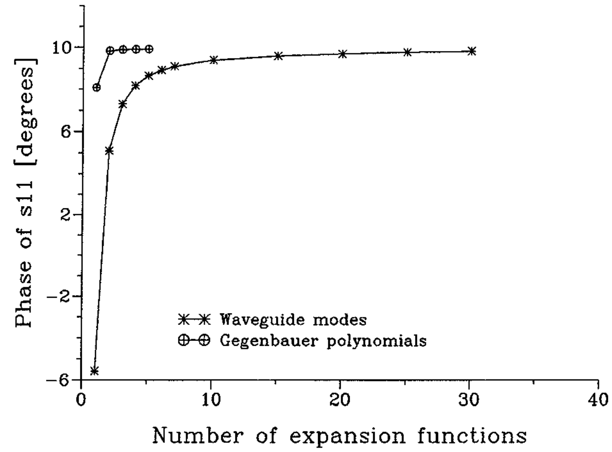

Fig. 3. A comparison of the numerical convergence of the analysis method for the two types of basis functions. The dimensions of the folded short are: a = 22.86 mm, $b1 = b2 = b3$ = 5.08 mm, t = 0.7 mm, $L1$ = 144.92 mm, $L3$ = 10.0 mm, and freq = 8.9 GHz.

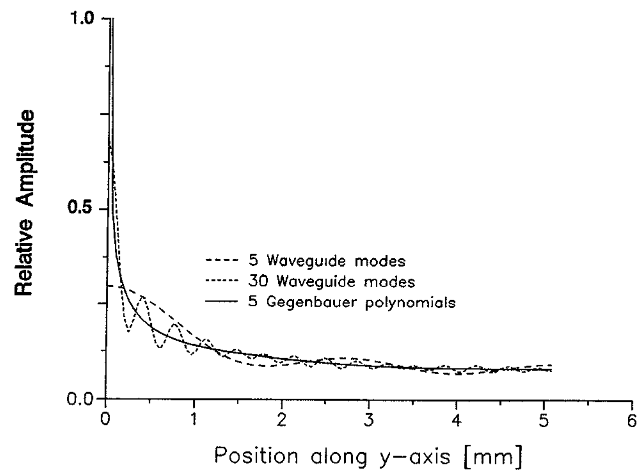

Fig. 4. The magnitude of the electric field in aperture A. The dimensions of the folded-short are a = 22.86 mm, $b1 = b2 = b3$ = 5.08 mm, t= 0.7 mm, $L1$ = 144.92 mm, $L3$ = 10.0 mm, and freq = 8.9 GHz.

$$g_b^i(\overline{r}) = \sin\left(\frac{\pi x}{a}\right)(1-\eta_b^2)^{-\frac{1}{3}} C_i^{(1/6)}(\eta_b), \quad i = 0, 2, 4, \ldots N \tag{10}$$

with

$$\eta_a = \frac{y - b_1}{b_1} \tag{11}$$

$$\eta_b = \frac{y + b_3 + t}{b_3} \tag{12}$$

$C_n^{(1/6)}(\eta)$ is a Gegenbauer polynomial of order n, parameter 1/6, and argument η. An explicit expression can be found in [4]. These Gegenbauer polynomials are orthogonal on the interval $\eta \in [-1, +1]$ with respect to the weighting factor $(1-\eta^2)^{-1/3}$, which exhibits the correct edge-condition singularity for the electric field at a 90° edge. Once the integrals in (2)–(6) have been evaluated (which have been done analytically) and the matrix equation in (1) solved, the reflection coefficient (or equivalent circuit, should it be required) of the folded short can be determined.

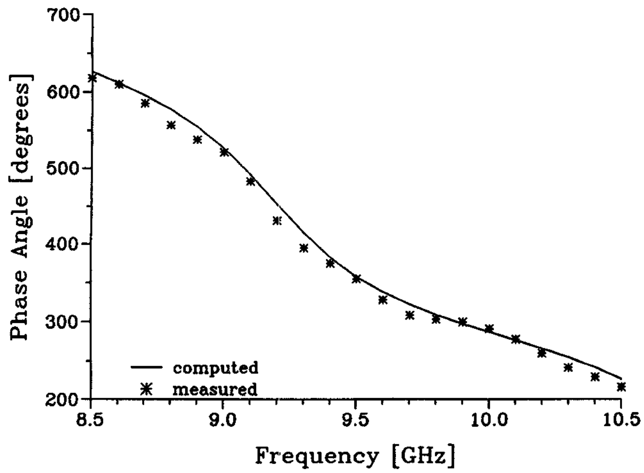

Fig. 5. A comparison of computed and measured phase data of the reflection coefficient of a folded waveguide short. The phase shown is the added phase with reference to a 150-mm-long standard waveguide short. The dimensions of the folded short are $a = 22.86$ mm, $b1 = b2 = b3 = 5.08$ mm, $t = 0.7$ mm, $L1 = 144.92$ mm, and $L3 = 60.0$mm.

III. Results

In Fig. 3, a comparison of the numerical convergence (for a single example) of the analysis method is shown for the two types of basis functions. It was found that two Gegenbauer polynomials provide an excellent approximation of the magnetic current (or equivalently the electric field) in the aperture, whereas more than 30 waveguide mode basis functions are needed to achieve the same accuracy in computed phase angle of the reflection coefficient of the folded short. In Fig. 4, the magnitude of the electric field in aperture A (see Fig. 2) is shown, respectively computed with five Gegenbauer polynomial basis functions, and five and 30 waveguide mode basis functions. It is obvious that the three computations are attempting to approximate the same aperture field and that the computation with five Gegenbauer polynomials is the most accurate (observe the expected edge-condition at $y = 0$). The theory was verified by comparing computed data to measurements of a folded short in half-height X-band waveguide. A comparison of computed and measured phase angle of the reflection coefficient is shown in Fig. 5. The agreement was found to be very good.

IV. Conclusion

An accurate moment method analysis of a folded E-plane waveguide short has been presented. The use of only two Gegenbauer polynomials provides very accurate answers for the reflection coefficient of such a folded short. The expected edge-condition behavior of the electric field at the edge singularities in the waveguide is inherently incorporated in the weighted Gegenbauer polynomial basis functions. The theory was verified by good agreement with measurements of a folded E-plane short in half-height X-band waveguide.

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